

# Flexible modelling with `inlabru`: A distance sampling case study

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Andy Seaton

University of St Andrews

# Acknowledgements

**Prof Janine Illian**, University of Glasgow

**Prof David Borchers**, University of St Andrews

**Dr Richard Glennie**, University of St Andrews

**Prof Finn Lindgren**, University of Edinburgh

**Dr Fabian Bachl**, University of Edinburgh

**Dr Rick Camp**, US Geological Survey

**Dr David Miller**, University of St Andrews

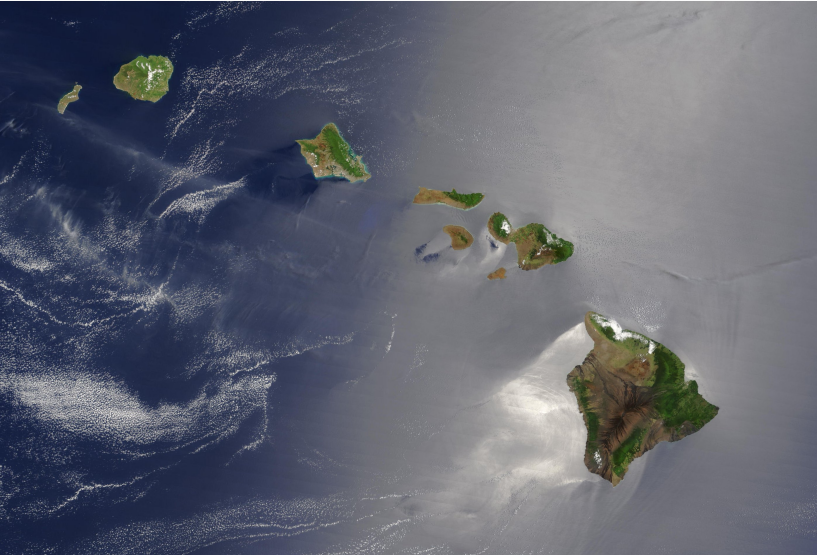
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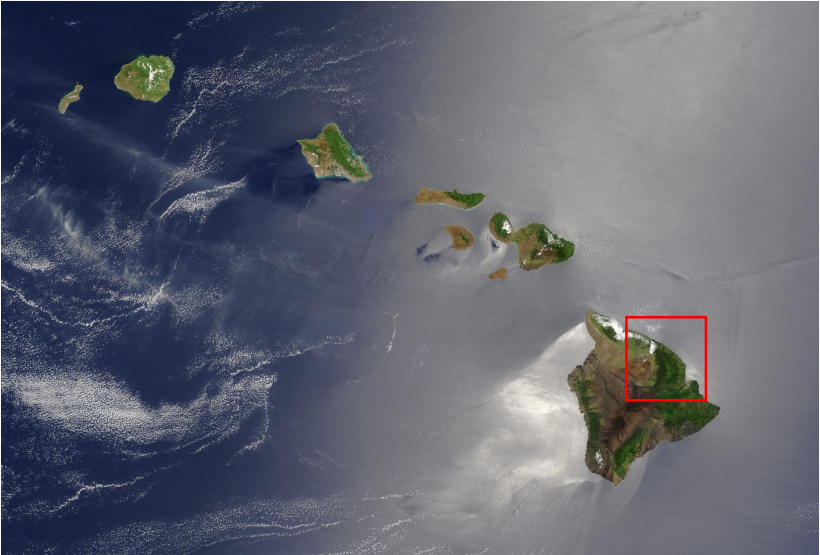
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1. Demonstrate how to fit distance sampling models in R-INLA/`inlabru`.
2. Highlight *general* features of `inlabru` that are applicable in a wide range of contexts (not just distance sampling).

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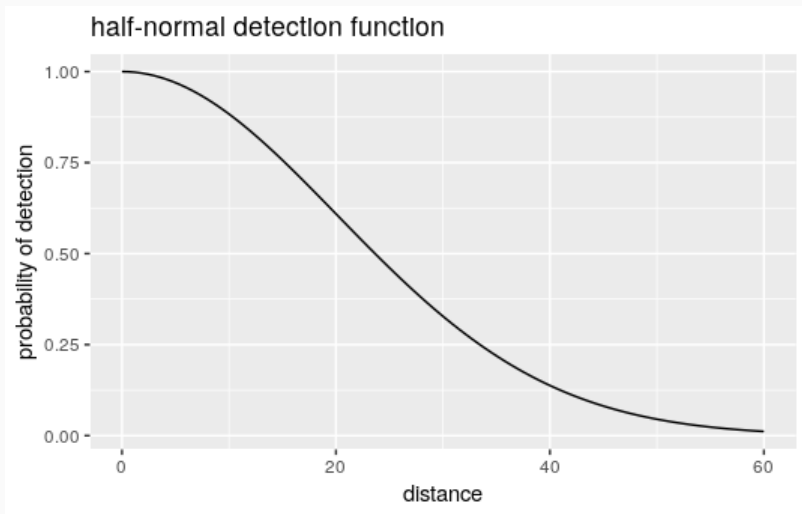
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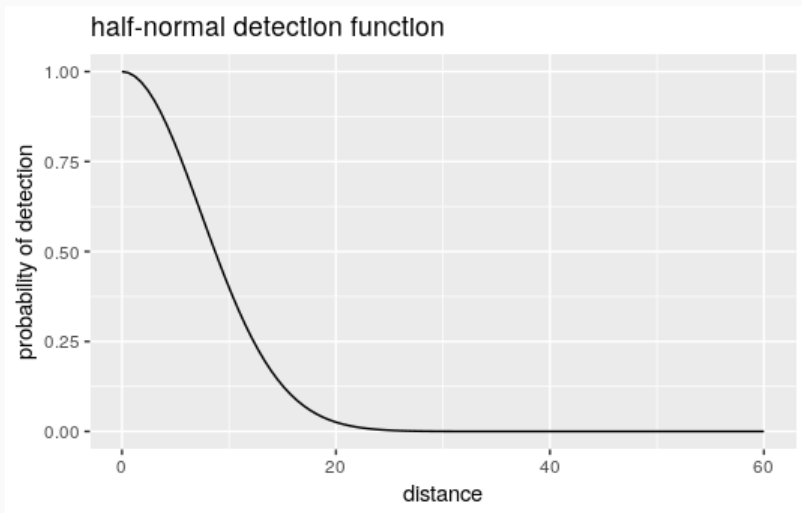
source: Jack Jeffrey, US Fish and Wildlife Service



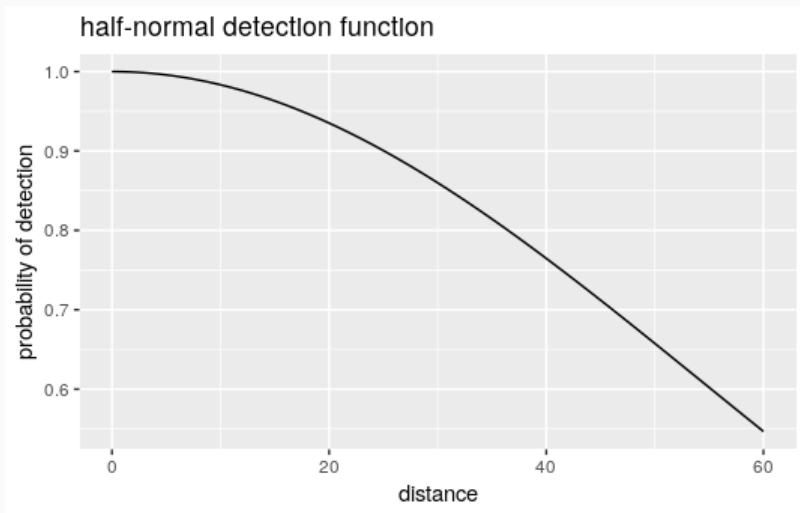
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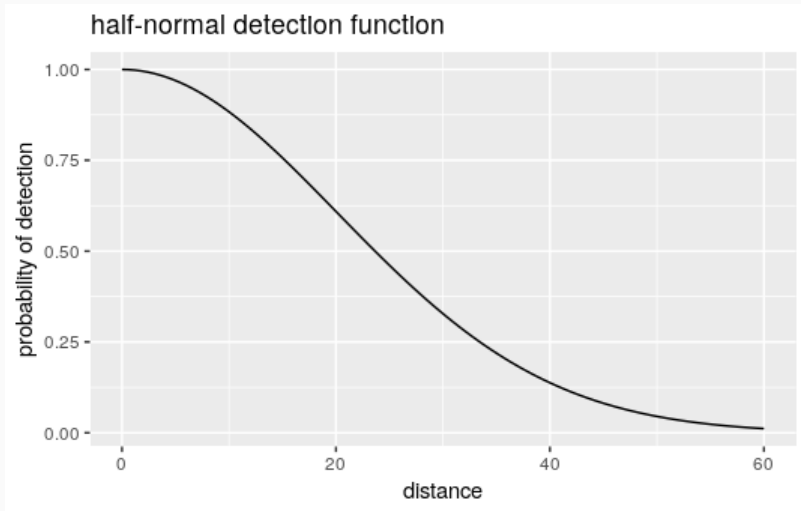
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Note the intercept assumed equal to 1

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Notable alternative: `unmarked` package (Fiske et al., 2011) does one stage maximum likelihood with a multinomial formulation (Royle et al., 2004) (distances + space must be discrete classes).

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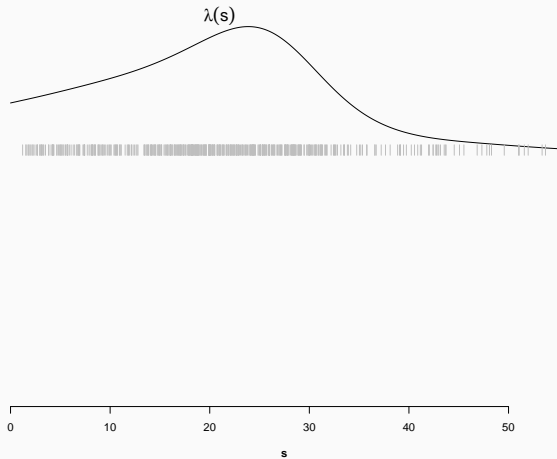
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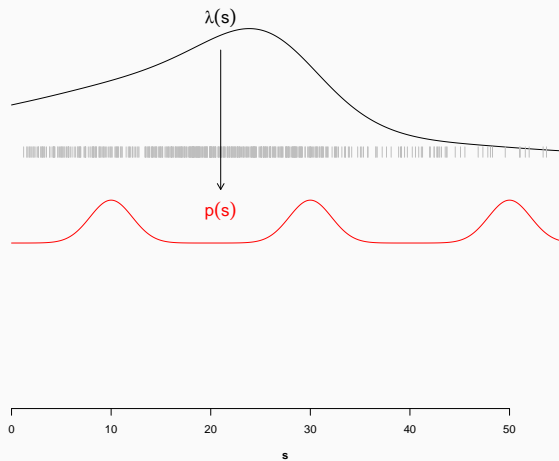
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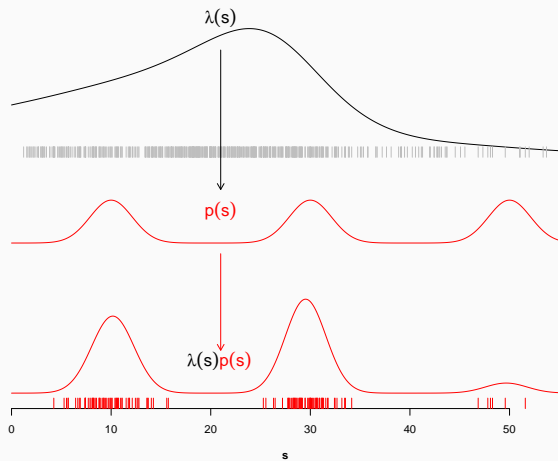
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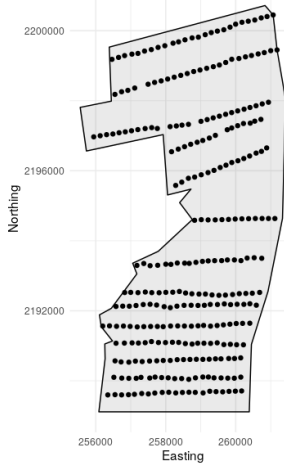
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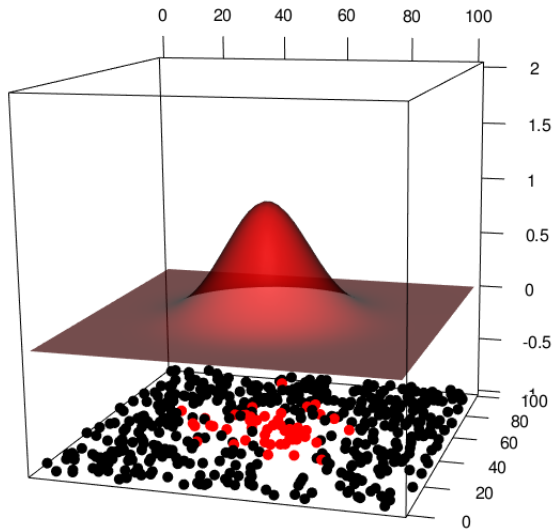


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This model is implemented in `inlabru` as a "cp" (Cox process) likelihood

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$2\pi r$  accounts for the increasing circumference as distance increases

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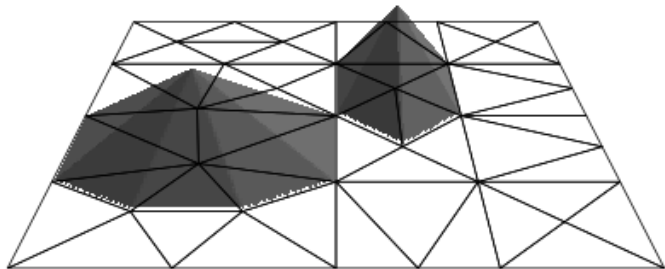
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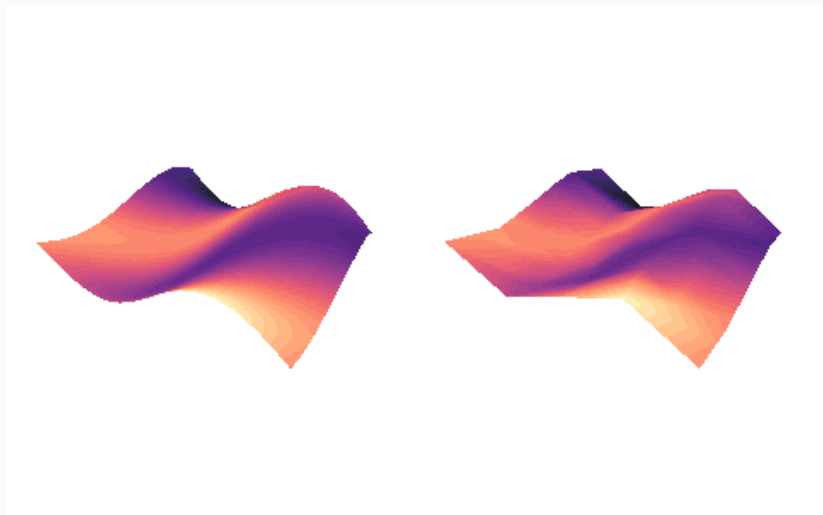
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we call this approach *iterated* INLA and it is implemented in `inlabru`  
(details in forthcoming paper...)

# Spatial Model





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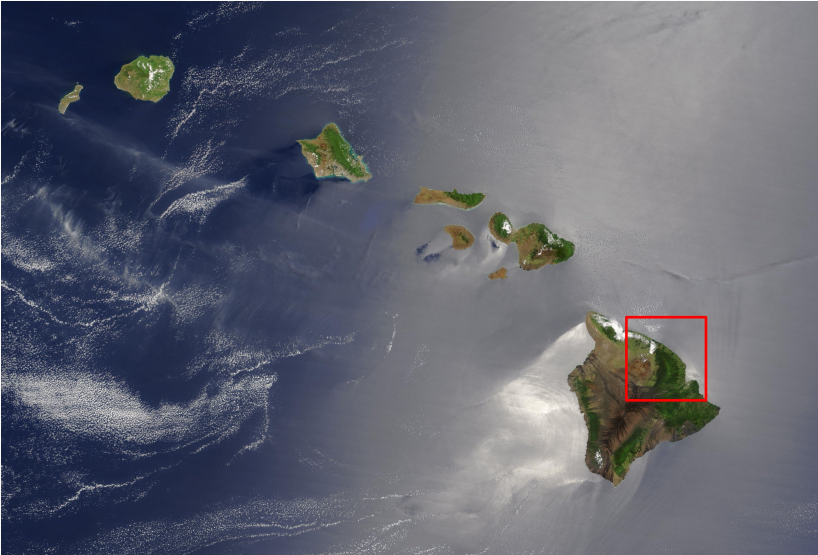
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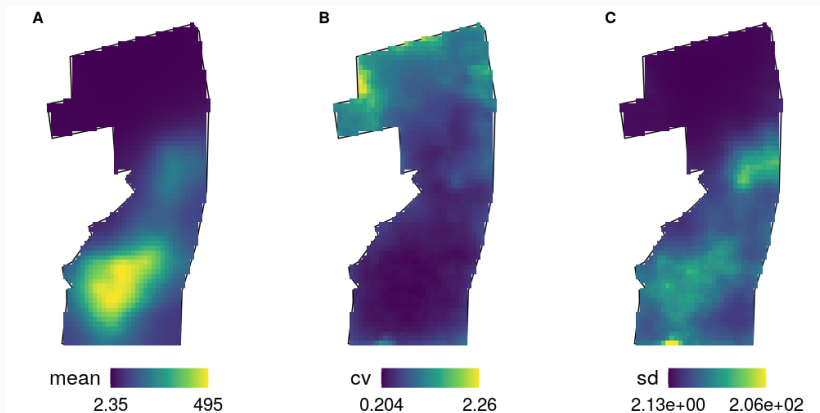
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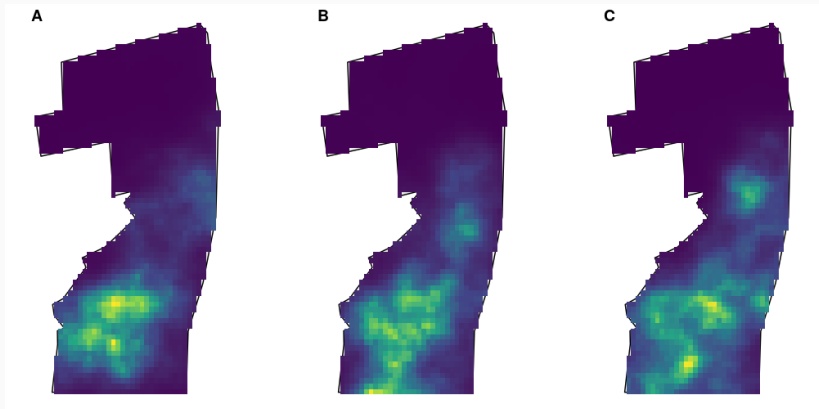


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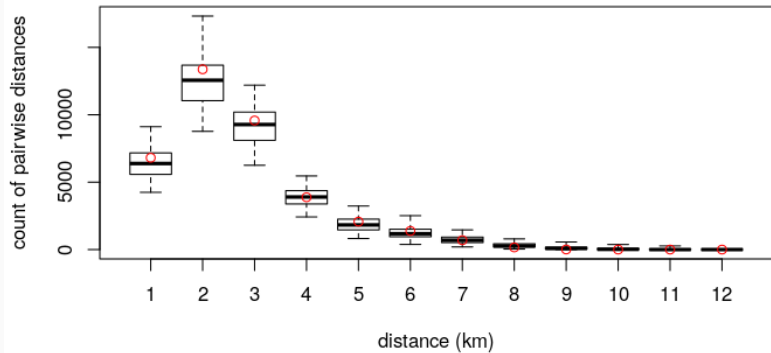
Summary statistics of the posterior intensity field

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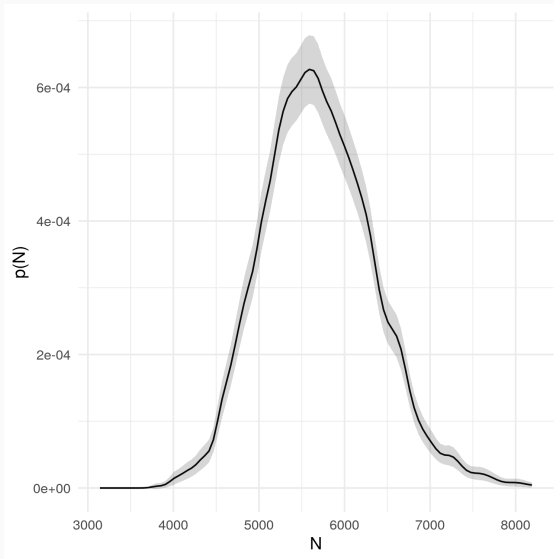
Three realisations of the posterior intensity field

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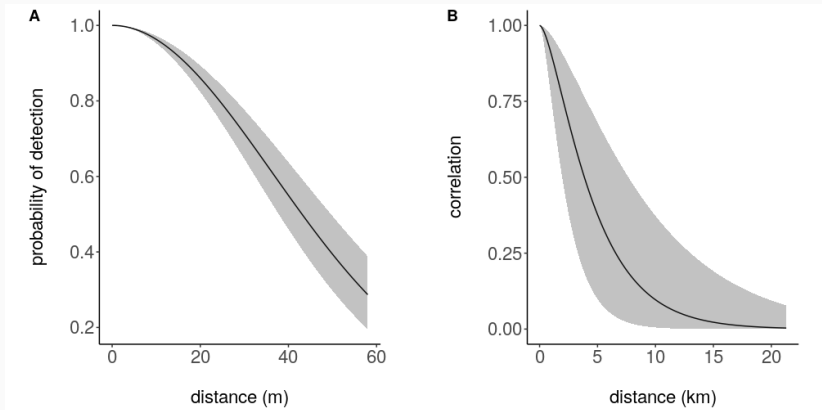
A “distance sampling adjusted” Ripley’s K-function

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Posterior of realised abundance  $\pm 2$  Monte Carlo standard errors

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Posterior detection function and Matérn covariance function

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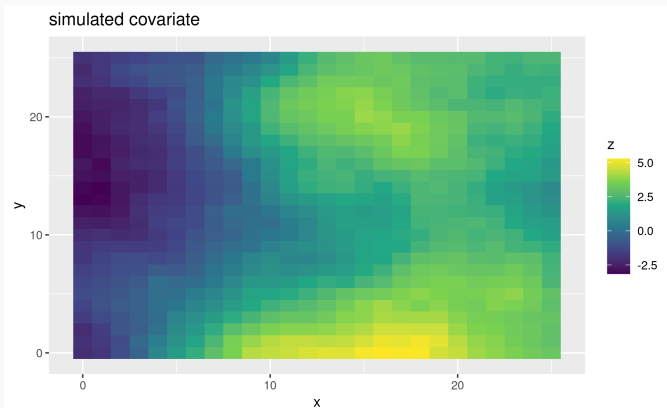
`inlabru` allows us to use `pnorm(...)`, for example.

# Non-linear model components in spatial ecology

## Functional responses

e.g. a “saturating” response:

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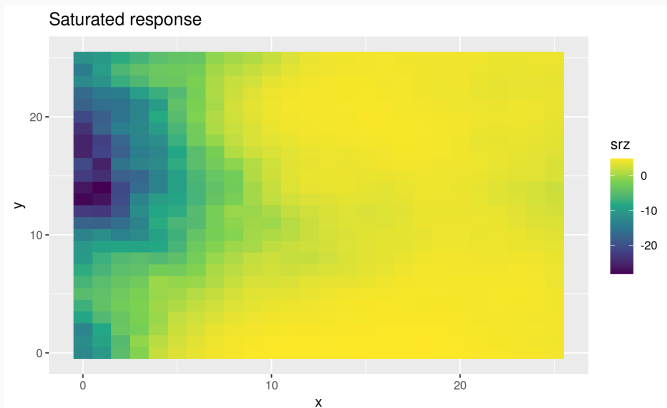


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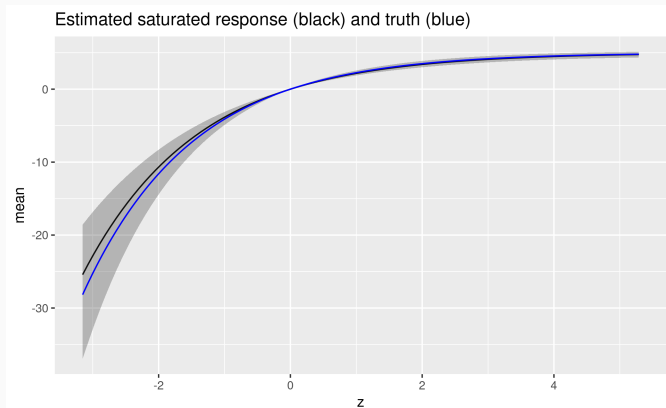


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- Trade-offs between resources (prey-choice literature)



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One more idea in the works:

- Level set Cox process - allow a mixture of random fields

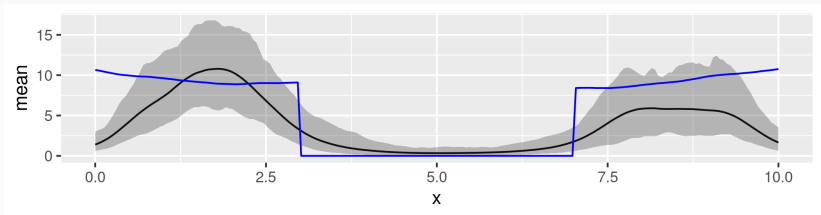
## A word of warning



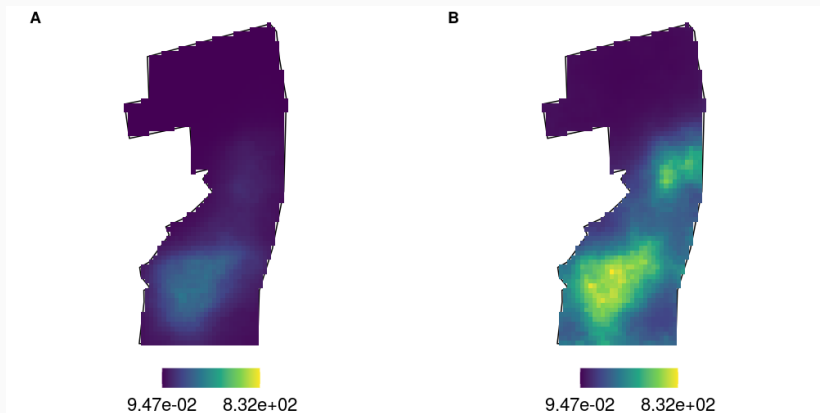
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# Extras!

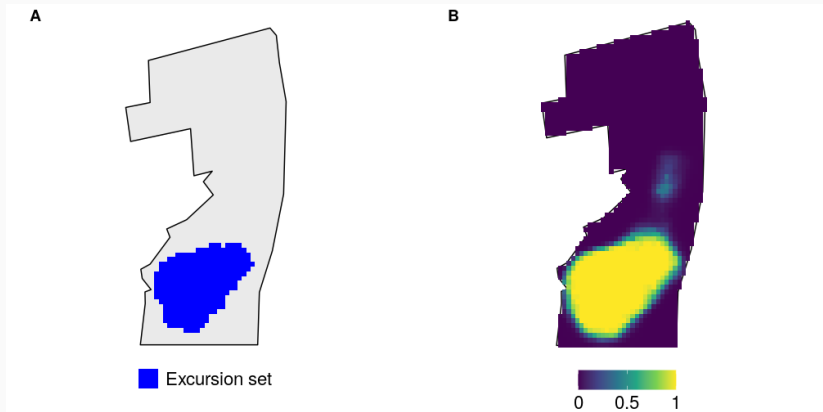


# Extras!



0.025 and 0.975 pointwise prediction quantiles for the posterior intensity field

# Extras!



Excursion set for 1 bird per hectare and 95% probability level and corresponding excursion function