# A new point pattern model for spatial capture recapture


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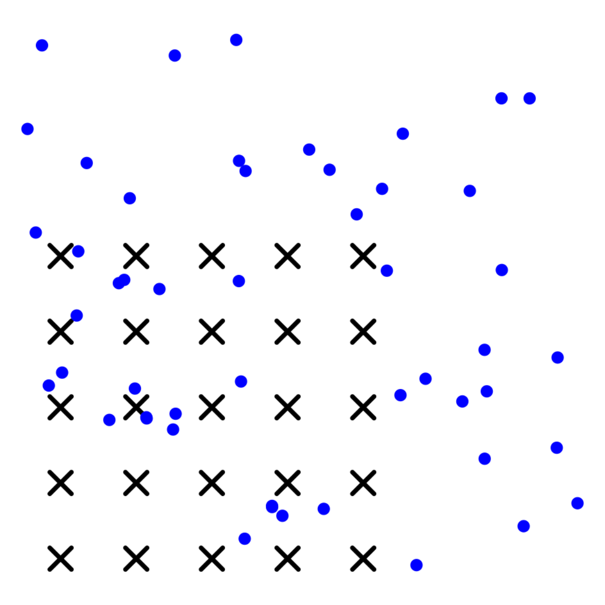
University of St Andrews

# Overview

1. Current spatial capture recapture point pattern models
2. Two new approaches, two simulation studies
3. Case study: Louisiana black bear data

# Spatial Capture Recapture



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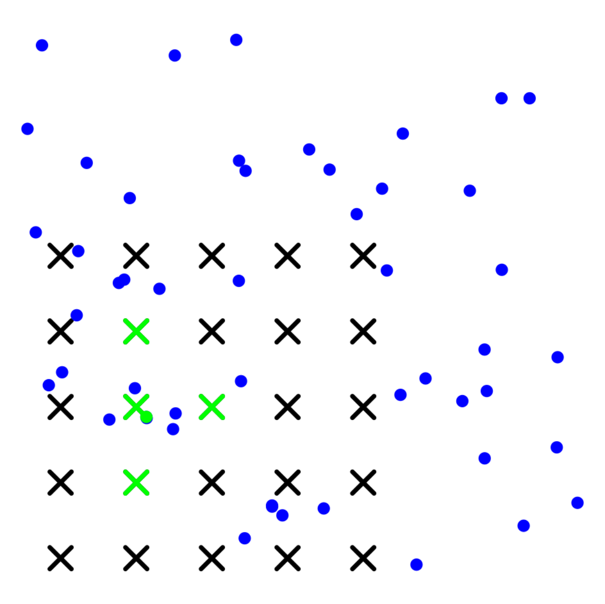

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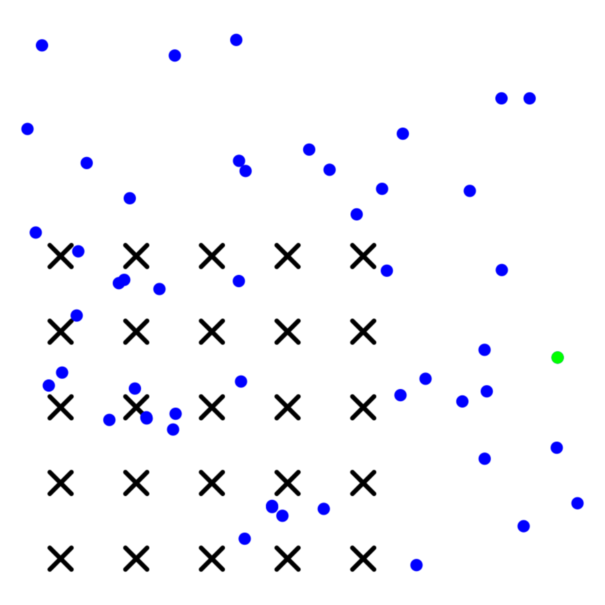
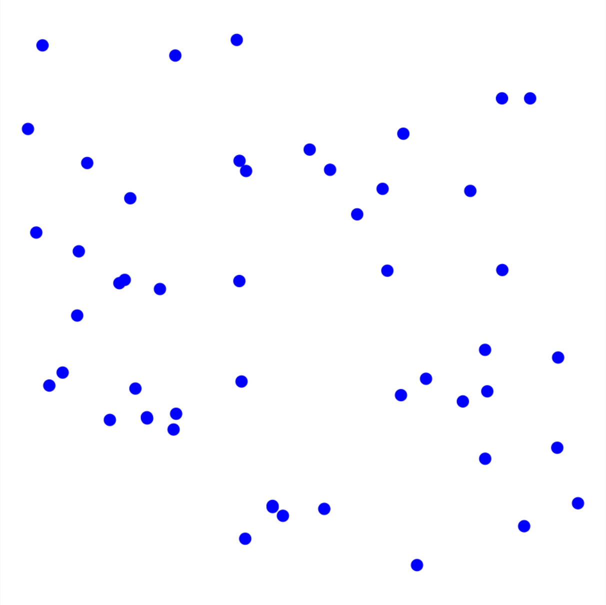


# Point Pattern Models



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- **Strauss process** for territorial species (Reich and Gardner, Environmetrics (2014))

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Implemented in Template Model Builder (TMB)
Kristensen et al., JSS (2016)

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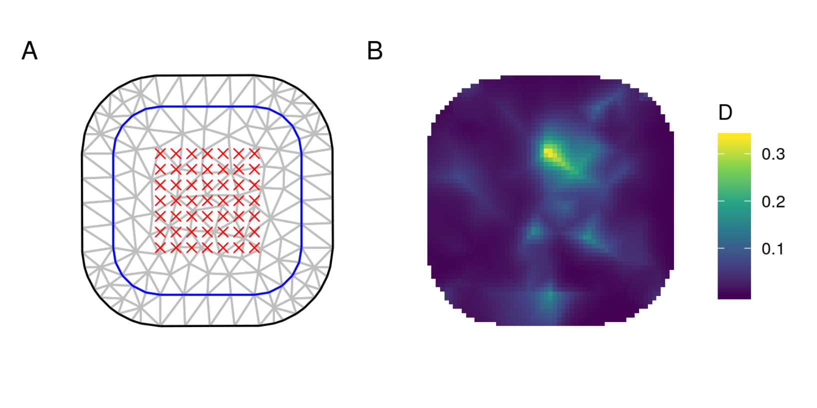

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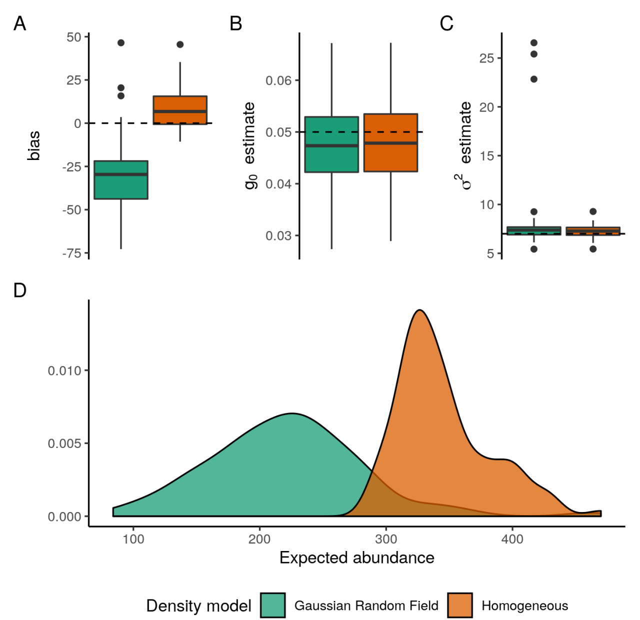

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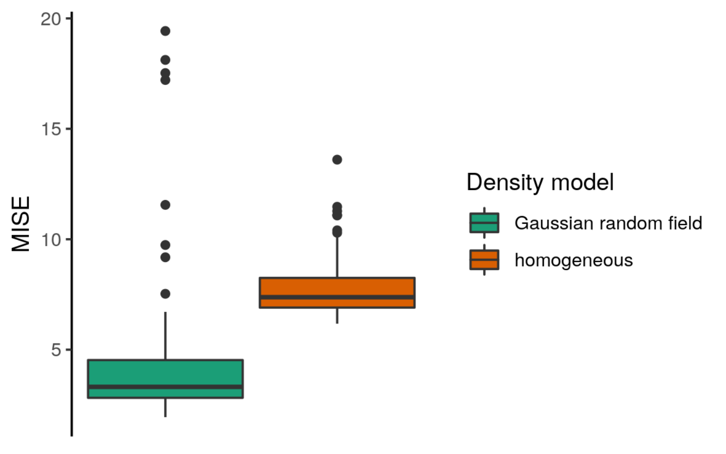

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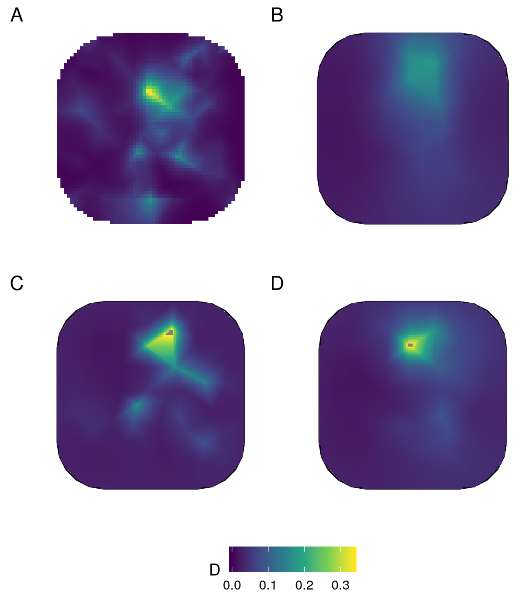

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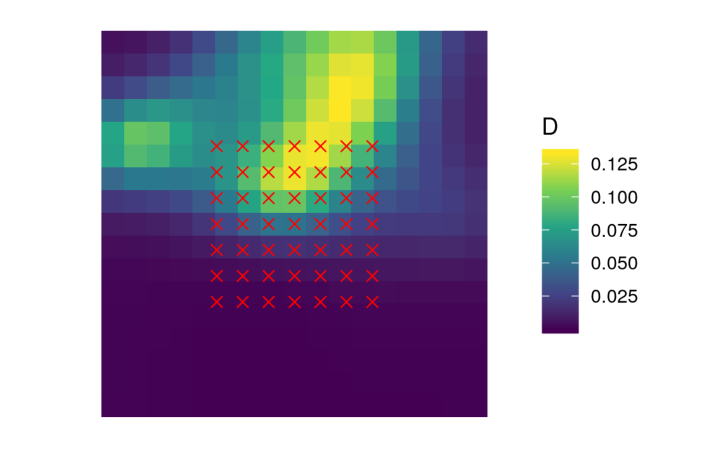
- Alternative to sparsity?

# Simulation Study - Thin Plate



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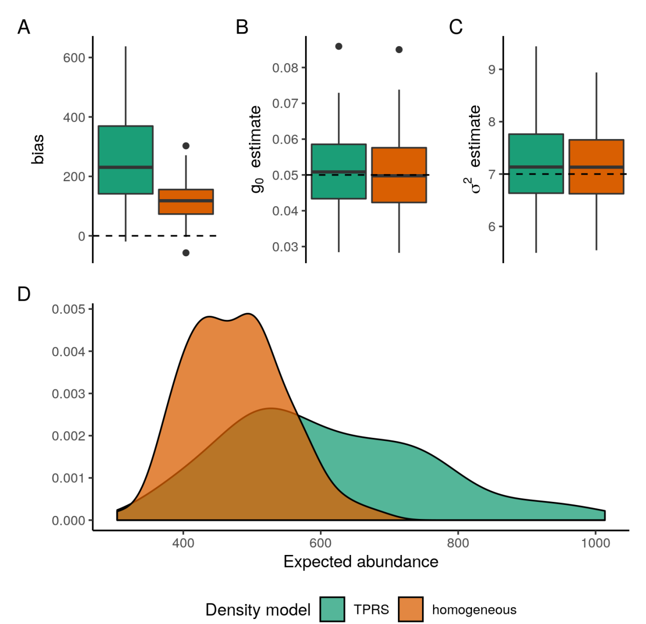

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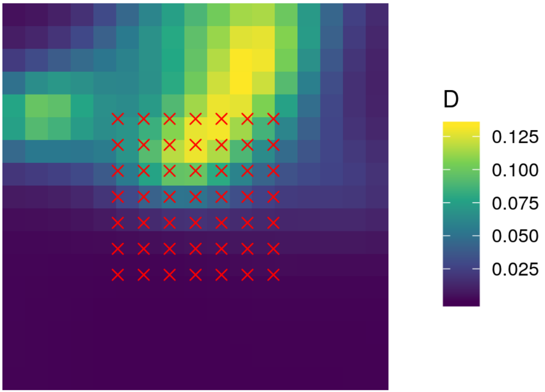

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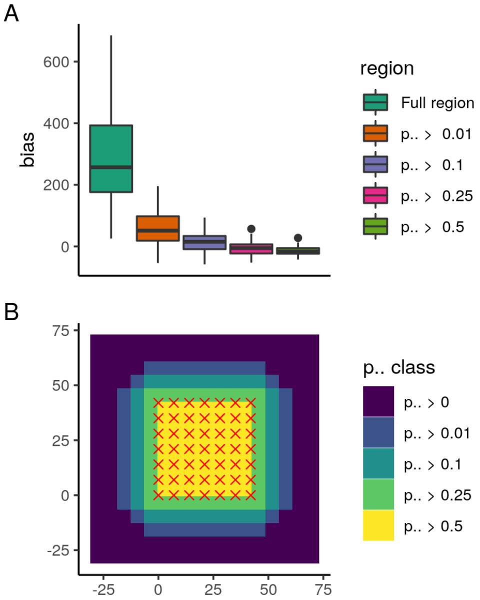

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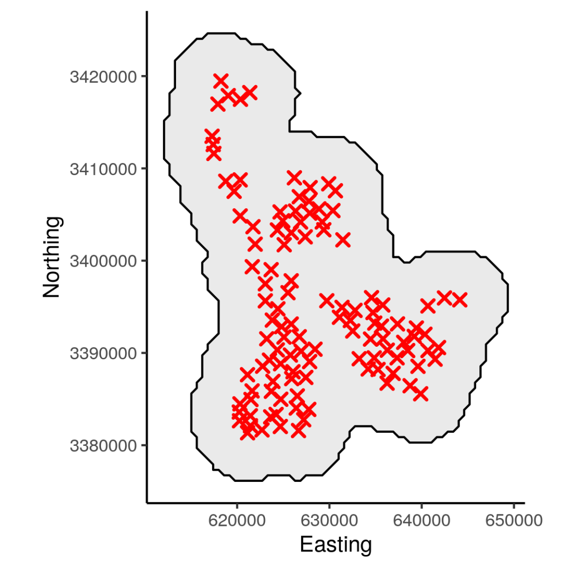
- Smoothing param not changing from starting value (argh!)

# Case Study - Louisiana Black Bears



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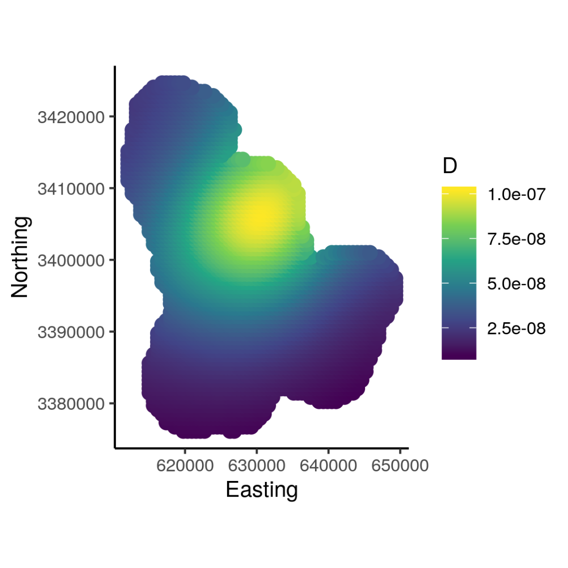

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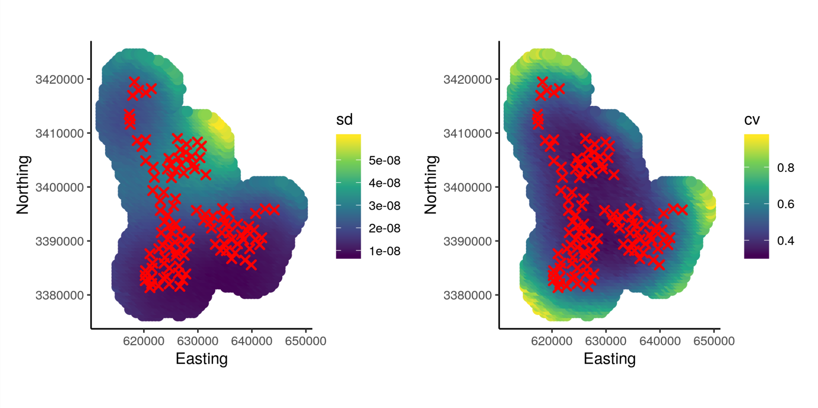

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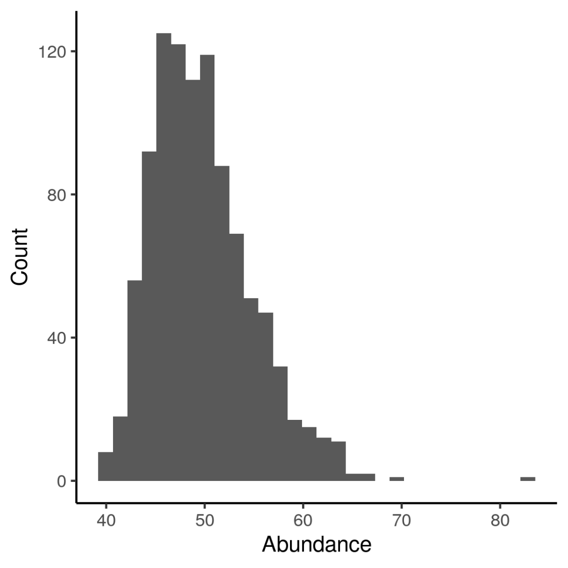

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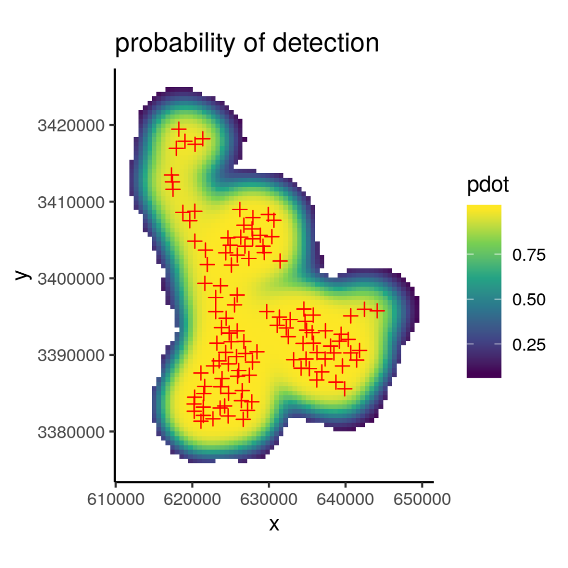

# Future Work

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4. 1D smooths on covariates

# Acknowledgements

Dr Ben Stevenson (University of Auckland)
Chandler and Clark, MEE (2014) (Black bear data)
C. Lowe, K. O'Connell (Black bear data)
Prof David Borchers (University of St Andrews)
Dr Richard Glennie (University of St Andrews)
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Thanks for listening!